Electric Potential and Field Mapping

Goals:

- To explore the electric potential surrounding two equally and oppositely charged conductors
- To identify equipotential surfaces/lines
- To show how the electric field and electric potential are related

Equipment:

- 25-V AC transformer
- Digital multimeter (DMM)
- Tray filled with water
- Plastic graph sheet
- Electrodes

Introduction:

Depending upon the situation at hand, we sometimes find it convenient to think about electrostatic phenomena in terms of electric fields and electric potentials. These two concepts are intimately related, and we will explore both of them in this lab exercise.

The **electric field** is defined to be the force *per unit charge* at a given point in space. The way we think about measuring the electric field is to take a very small positive *test* charge, q_0, and measure the force on it due to the *source* charges around it. At each point P in space, we measure the force on our test charge, and then simply define the electric field to be the force exerted on our test charge divided by q_0:

$$\vec{E}(\text{at point } P) \equiv \frac{\vec{F}_{\text{on } q_0}(\text{at point } P)}{q_0}. \qquad \text{(Eq. 1)}$$

A force is always a vector quantity, and so the electric field must also be a vector quantity.

Now, we can turn this equation around and ask a different question. If we suppose that we already know, at a given point P in space, what the magnitude and direction of the electric field there is, what force would a charge q experience if it were placed at that point? From the definition of the electric field in Eq. 1, we see that at any point in space

$$\vec{F}_{\text{on } q} = q\vec{E}. \qquad \text{(Eq. 2)}$$

To summarize, Equation 1 tells us how to measure the electric field at a point in space: we simply measure the force exerted on a test charge, and divide the force vector by that charge. Equation 2, then, tells us how to find the force exerted *by* an electric field on a charged particle: we simply multiply the electric field vector by the amount of charge on our particle.

Using the methods described above, we could find the electric field created by a single charged particle. For a single source charge q, the *magnitude* of the electric field around it is given by

$$E = k\frac{|q|}{r^2}, \qquad \text{(Eq. 3)}$$

where r is just the distance from the source charge q to the point in question. For the *direction* of the electric field, we have a simple rule: the electric field due to a point charge always points directly *away* from a positive charge and always directly *toward* a negative charge. In the event that there are multiple source charges present, then we can find the *net* electric field at a particular point in space by finding the *vector* sum of the electric fields at that point due to each individual charge.

Now, Equations 1–3 provide us with enough information to determine the forces between any number of charges in the universe. Combined with Newton's second law ($\vec{F} = m\vec{a}$), we should be able to calculate how the charges will move due to these interactions. As was the case in classical mechanics, however, we often find that we can understand these interactions more simply if we set aside forces & kinematics and focus our attention instead upon *energy* and *energy conservation*. Energy comes in many forms, and the form of interest to us here is **potential energy**, U. Because the electric force is a conservative force, we can define a potential energy associated with the interaction between different charged particles. For instance, two oppositely charged particles will be strongly attracted to each other when they are close together. You can *increase* their potential energy by pulling them apart (you must do work in this process). When you let the charges go, they will accelerate toward each other, and their potential energy will *decrease* while their kinetic energy increases.

So, we're interested in discussing the electrical potential energy associated with systems of charged objects. To make matters simpler, we're going to play a similar game we played with the electric field. The electric field, again, is the *force per unit charge* that a test charge experiences at a particular point in space. We will now define the **electric potential difference**, ΔV, to be the difference in *electric potential energy per unit charge* that a test charge q_0 would have between two particular points in space. In contrast with the electric field, the electric potential is a scalar quantity (like temperature, it only has a magnitude, but no direction associated with it.) So, in an equation form, the electric potential difference ΔV between two points in space is defined by

$$\Delta V = V_B - V_A \equiv \frac{\Delta U_{\text{of } q_0}}{q_0} = \frac{U_{\text{of } q_0}(\text{at point B}) - U_{\text{of } q_0}(\text{at point A})}{q_0}. \qquad \text{(Eq. 4)}$$

We can again reverse our point of view: if we suppose that we already know what the electric potential at every point in space is, then we would know how much potential energy a charged particle would gain or lose if we moved it from one point to another:

$$\Delta U_{\text{of } q} = q\Delta V. \qquad \text{(Eq. 5)}$$

Much as we asked what the electric field created by a single charge is, we can now ask what the electric potential around a single charged particle is. We find that it is simply

$$V = k\frac{q}{r}. \qquad \text{(Eq. 6)}$$

You may recall from an earlier physics course that we can choose any point(s) we want to be the location where the potential energy is zero. We almost always find it convenient to say that the potential energy is zero when very, very far away from any source charges (*i.e.*, at infinity). This convention has been used to come up with the expression for Eq. 6. Because of this convention, the potential around a source charge has a positive value if the source charge is positive, and a negative value if the source charge is negative. When there are multiple source charges present, we simply find the total electric potential at a point in space by adding together (paying attention to the sign) the potentials due to each individual charge at that point in space.

So, the electric field is defined to be the force per unit charge, and the electric potential is defined to be the potential energy per unit charge. Because of the relationship between forces and their associated potential energies, we have a very powerful relationship between the electric field and the electric potential, namely:

$$E = -\frac{\Delta V}{\Delta s}, \qquad \text{(Eq. 7)}$$

which tells us that the electric field is directly related to the *rate of change* of the electric potential from point to point in space. The more drastically the electric potential changes from one point to another, the greater the magnitude of the electric field pointing along that displacement. Conversely, if the electric potential doesn't change at all from one point to another (*i.e.*, the electric potential is constant), then the electric field cannot point in the direction of that displacement at all (we could say that the component of the electric field in that direction must be zero.) This is at the heart of the notion of an **equipotential** line or surface, which you will be exploring today.

To be specific, the line connecting different points that are all at the same electric potential is known as an **equipotential line**. From Equation 7 and from the discussion above, we can see that the electric field must never point along an equipotential line. In other words, *the electric field will always be perpendicular to an equipotential line*. The negative sign in Eq. 7 tells us that the electric field always points in the direction of *decreasing* electric potential.

In today's lab activity, we will be interested in seeing how charged objects affect the space around them. To do so, we often wish to create a figure or an image which illustrates all the available information about the charges and the space around them. We typically draw **electric field lines** to tell us about the electric field around the charges. At any point along a field line, the electric field must be tangent (aligned) with the field line. The arrows give us an indication of the direction of the field. Where the field lines are closer together, the electric field is stronger. Where the field lines are spread widely apart, the electric field is relatively weak. So, we can convey a lot of information simply by indicating the location of a charge and drawing several field lines around it.

We can also convey complementary information by simultaneously drawing the equipotential lines around our charges. (Again, the equipotential lines are lines which connect different points in space that are all at the same electrical potential *V*.) In Figure below, we show several examples of different configurations of charges and how their electric field line/equipotential line diagrams should appear.

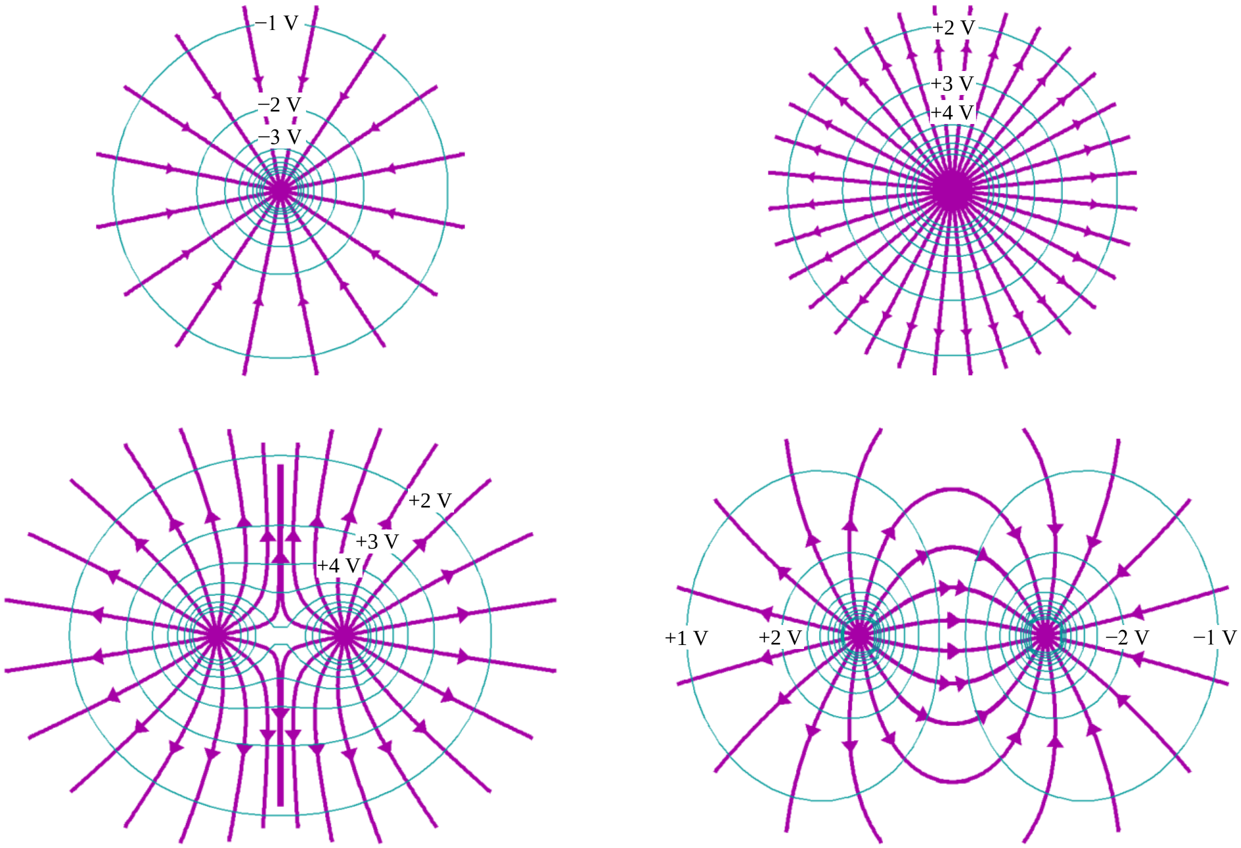

Figure 1 Electric field line and equipotential line diagrams for several different systems of charges. Electric field lines are depicted with a bold line with arrows indicating the direction of the electric field. The equipotential lines are depicted by the fainter line. *Top-left*: A single, isolated particle with a charge $-q$. *Top-right*: A single isolated particle with charge $+2q$. *Bottom-left*: Two positive charges both with charge $+q$, separated by a small distance along the horizontal axis. *Bottom-right*: Two particles, one with charge $+q$, the other with a charge $-q$, separated by a small distance along the horizontal axis. Note that in all four of these diagrams, *the electric field lines are always perpendicular to the equipotential lines*.

These are the major features of our field and potential models of static electricity. On top of this, we've also explored how some materials behave differently in the presence of charges, and created two general categories of materials: **insulators**, which do not allow charges to move easily through or along them, and **conductors**, which do easily allow charges to flow through or along them. This defining property of conductors has some special consequences in terms of our field and potential models:

- Excess charge placed on a conductor moves to the exterior surface of the conductor, almost instantaneously.
- The electric field inside a conductor in electrostatic equilibrium is zero.
- Electric field lines meet the surface of a conductor in equilibrium at right angles.
- Every point on or within a conductor in equilibrium is at the same electric potential.

The last two observations about conductors will play a role during the activities and analysis of today's lab.

Name: ____________________________ Sect.: _______

Name: ____________________________ Name: ____________________________

Directions:

Set up the apparatus as in Figure 2 below. The plastic graph sheet should be on the bottom of the tray, and the tips of the electrodes should touch the plastic sheet (not the tray itself). Fill the tray with water (if it is not already filled). Arrange the tips of the electrodes so that they rest at different points along the same bold horizontal line (which you can now call your *x*-axis). For the sake of convenience, you'll probably want to arrange the tips of the electrodes so that they each sit at distinct bold vertical lines.

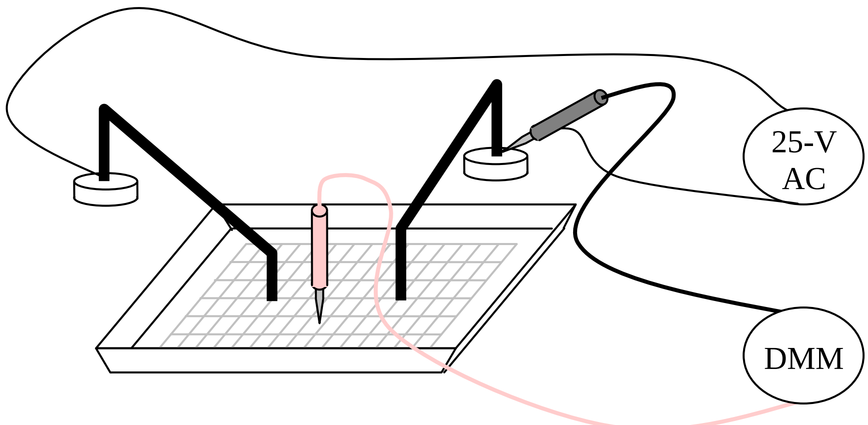

Figure 2 Setup used to determine the electric potential around two equally but oppositely charged conductors

If it is not already connected, connect each electrode to a different terminal of the transformer. For our purposes, we can think of the transformer as simply serving to keep a steady and reliable charge on each electrode. Connect the black lead of the digital multimeter (DMM) to one terminal of the transformer and use the red lead as a probe to measure the potential. Set the DMM to read AC on the 200-V scale.

You will be using this DMM to measure **potential differences** between points. Recall that we can only measure *differences* in the electric potential, and not the electric potential itself. When used this way, the DMM acts as a **voltmeter**, a device which measures the potential difference between the tips of its probes. In other words, the DMM (assuming the probes are connected in the standard way) will display the value of $\Delta V = V_{\text{red}} - V_{\text{black}}$, as measured in the standard S.I. unit of electric potential, volts.

Now, we have decided to connect the black probe of our DMM to the same terminal of the transformer as one of the electrodes. By placing the black probe in **electrical contact** this terminal, it will be at the same electrical potential as that terminal (and any other conductor in contact with it). This happens because charges are free to move across two conductors in contact, effectively making them behave as one larger conductor. We also stated that every point on or within a conductor in equilibrium must be at the same potential. Combining these observations, we see that the black lead must be at the same potential as the transformer terminal it is in contact with.

Again, the DMM only measures the potential difference $\Delta V = V_{red} - V_{black}$. So, if we place the red probe in contact with the same electrode to which the black probe is connected, the meter should show no potential difference (a voltage of ‘0’) — you should verify this for yourself. At any other point, the red probe will measure the electric potential *relative* to the electric potential at the black probe. In this way, we will use the red probe to map the electric potential at any point we wish around both electrodes.

<u>Activity 1: Plotting the equipotential lines</u>

1. You will be measuring the electric potential at particular points in the water-filled tray, and then recording these values on a similar piece of graph paper (found as the last page of this lab). To begin, first indicate on your graph paper where the electrodes are located.

2. Place the red probe halfway between the two electrodes (on your *x*-axis), and record its position and the electric potential at that location on your graph paper.

3. Now place the probe a few centimeters away from your first point along a vertical gridline, and move the probe around until the potential is the same as in step 2. This is a second point on the equipotential line. Record its location on your graph paper.

4. Continue to move the probe a few centimeters at time, finding points that have the same potential as in step 2, and recording the location of these points on your graph paper. Be sure to move your probe to points above and below your *x*-axis. Once you have enough points to have a good idea of how the equipotential line looks, you can stop recording points for this line.

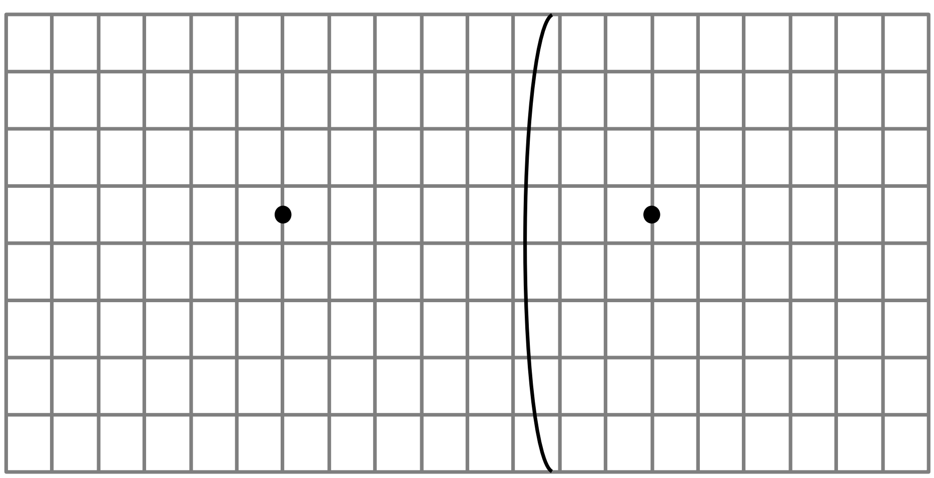

Figure 3 Graph paper after plotting one equipotential line

5. On your graph paper, draw your best estimate of what the equipotential line for this value of the electric potential looks like. You should not just “connect the dots:” if it looks like the equipotential line curves a bit, you should try your best to draw a smoothly curving line. Be sure to label the equipotential line with the value of the potential along that line. At this point, your graph paper should probably look something like Figure 3.

6. Now, follow essentially the same procedure as outlined in steps 3–5 above for *at least* six more equipotential lines, each with a different value of the electric potential. Your goal is to have a reasonably accurate "map" of the equipotential lines throughout the region between and around the electrodes.

7. As was discussed in the introduction, electric field lines should always be perpendicular to equipotential lines. In another color of pen or pencil (if available), draw at least five electric field lines on your graph paper and be sure to draw arrows to indicate the direction of the electric field somewhere on each line.

Analysis

Q1. If we assume that the two electrodes in our experiment each have a static (non-changing) net charge on them, how could you use your plot of equipotential lines to determine which electrode is more positively charged?

Q2. Suppose you placed an electron ($q = -1.60 \times 10^{-19}$ C) at a random point on one of your equipotential lines. How much work would it require to push that electron 10 cm, precisely along that equipotential line?

Q3. Suppose you instead decide to move a single electron from the "positive" electrode to the other electrode. Does the amount of work required to push the electron depend upon what path you choose to go between the two electrodes? Why or why not?

Q4. Now estimate (based on your actual data) how much work would be required to move an electron from the "positive" electrode to the other.

Q5. Which of the following can actually be physically measured: the value of the electric potential at a point in space, or the difference in the electric potential between two points in space? Explain your response.

Q6. When viewed from above, your two conducting electrodes had circular cross-sections where they intersected the water. Suppose that the electrodes you used were replaced by different electrodes which had cross-sections like those shown below. Sketch what the electric field and equipotential lines would look like for these electrodes.

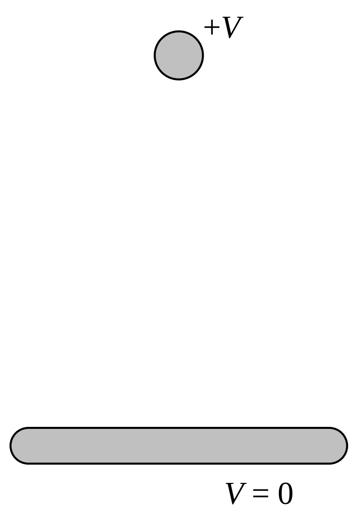

Figure 4 Alternate conducting electrodes